OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4725

Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4725
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Wednesday 19 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 The matrices **A**, **B** and **C** are given by $\mathbf{A} = (2 \quad 5)$, $\mathbf{B} = (3 \quad -1)$ and $\mathbf{C} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$. Find

(i) $2\mathbf{A} + \mathbf{B}$, **[2]**

(ii) $\mathbf{AC}$, **[2]**

(iii) $\mathbf{CB}$. **[3]**

2 The complex numbers z and w are given by $z = 4 + 3\mathrm{i}$ and $w = 6 - \mathrm{i}$. Giving your answers in the form $x + \mathrm{i}y$ and showing clearly how you obtain them, find

(i) $3z - 4w$, **[2]**

(ii) $\dfrac{z^*}{w}$. **[4]**

3 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 2$, and $u_{n+1} = 2u_n - 1$ for $n \geqslant 1$. Prove by induction that $u_n = 2^{n-1} + 1$. **[4]**

4 Given that $\sum_{r=1}^{n}(ar^3 + br) \equiv n(n-1)(n+1)(n+2)$, find the values of the constants a and b. **[6]**

5 Given that **A** and **B** are non-singular square matrices, simplify

$$\mathbf{AB}(\mathbf{A}^{-1}\mathbf{B})^{-1}.$$ **[3]**

6 **(i)** Sketch on a single Argand diagram the loci given by

(a) $|z| = |z - 8|$, **[2]**

(b) $\arg(z + 2\mathrm{i}) = \frac{1}{4}\pi$. **[3]**

(ii) Indicate by shading the region of the Argand diagram for which

$$|z| \leqslant |z - 8| \quad \text{and} \quad 0 \leqslant \arg(z + 2\mathrm{i}) \leqslant \tfrac{1}{4}\pi.$$ **[3]**

7 **(i)** Write down the matrix, **A**, that represents a shear with x-axis invariant in which the image of the point (1, 1) is (4, 1). **[2]**

(ii) The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{3} \end{pmatrix}$. Describe fully the geometrical transformation represented by **B**. **[2]**

(iii) The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 2 & 6 \\ 0 & 2 \end{pmatrix}$.

(a) Draw a diagram showing the unit square and its image under the transformation represented by **C**. **[3]**

(b) Write down the determinant of **C** and explain briefly how this value relates to the transformation represented by **C**. **[2]**

8 The quadratic equation $2x^2 - x + 3 = 0$ has roots α and β, and the quadratic equation $x^2 - px + q = 0$ has roots $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$.

(i) Show that $p = \frac{5}{6}$. **[4]**

(ii) Find the value of q. **[5]**

9 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} a & -a & 1 \\ 3 & a & 1 \\ 4 & 2 & 1 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of $\mathbf{M}$. **[3]**

(ii) Hence find the values of a for which $\mathbf{M}^{-1}$ does not exist. **[3]**

(iii) Determine whether the simultaneous equations

$$\begin{aligned} 6x - 6y + z &= 3k, \\ 3x + 6y + z &= 0, \\ 4x + 2y + z &= k, \end{aligned}$$

where k is a non-zero constant, have a unique solution, no solution or an infinite number of solutions, justifying your answer. **[3]**

10 **(i)** Show that $\frac{1}{r} - \frac{2}{r+1} + \frac{1}{r+2} \equiv \frac{2}{r(r+1)(r+2)}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)}.$$ **[6]**

(iii) Show that $\sum_{r=n+1}^{\infty} \frac{2}{r(r+1)(r+2)} = \frac{1}{(n+1)(n+2)}$. **[3]**

1	**(i)**	$\begin{pmatrix}7 & 9\end{pmatrix}$	B1B1 **2**	Each element correct **SC** (7,9) scores B1
	(ii)	$\begin{pmatrix}18\end{pmatrix}$	B1* depB1 **2**	Obtain correct value Clearly given as a matrix
	(iii)	$\begin{pmatrix}12 & -4\\ 6 & -2\end{pmatrix}$	M1 A1 A1 **3** **7**	Obtain 2×2 matrix Obtain 2 correct elements Obtain other 2 correct elements
2.	**(i)**	$-12+13\text{i}$	B1B1 **2**	Real and imaginary parts correct
	(ii)	$\frac{27}{37}-\frac{14}{37}\text{i}$	B1 M1 A1 A1 **4** **6**	z^* seen Multiply by w^* Obtain correct real part or numerator Obtain correct imaginary part or denom. Sufficient working must be shown
3			B1* M1* A1* depA1 **4** **4**	Establish result true for $n = 1$ or 2 Use given result in recurrence relation in a relevant way Obtain $2^n + 1$ correctly Specific statement of induction conclusion
4		***Either***	B1	Correct value for $\sum r$ stated or used
			M1	Express as sum of two series
		$\frac{a}{4}n^2(n+1)^2+\frac{bn}{2}(n+1)$	A1	Obtain correct unsimplified answer
			M1	Compare coefficients or substitute values for n
		$a=4 \quad b=-4$	A1 A1 **6**	Obtain correct answers
		Or		
			M1	Use 2 values for n
		$a+b=0 \quad 4a+b=12$	A1 A1	Obtain correct equations
			M1	Solve simultaneous equations
		$a=4 \quad b=-4$	A1 A1	Obtain correct answers
			6	
5			B1	$(\mathbf{A}^{-1})^{-1} = \mathbf{A}$ seen or implied
			M1	Use product inverse correctly
		$\mathbf{A}^2$	A1cao **3**	Obtain correct answer
			3	

6	(i)	(a)	B1*		Vertical line
			depB1	**2**	Clearly through (4, 0)
		(b)	B1		Sloping line with +ve slope
			B1		Through (0, -2)
			B1ft	**3**	Half line starting on y-axis 45° shown convincingly
	(ii)		B1ft		Shaded to left of their **(i) (a)**
			B1ft		Shaded below their **(i) (b)** must be +ve slope
			B1ft	**3**	Shaded above horizontal through their (0, -2)
					NB These 3 marks are independent, but 3/3 only for fully correct answer.
				8	

7	(i)	$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$	B1 B1	**2**	Each column correct
	(ii)		B1*		Enlargement or stretch in x **and** y axes
			depB1	**2**	Scale factor $\sqrt{3}$
	(iii) (a)		B1		$(2,0),(6,2)$ indicated
			B1		(8, 2) seen
			B1	**3**	Accurate diagram, including unit square
	(b)	det**C** = 4	B1		Correct value found
			B1	**2**	Scale factor for area
				9	

8	(i)	***Either***			
		$\alpha+\beta=\frac{1}{2}, \alpha\beta=\frac{3}{2}$	B1		State or use both correct results in (i) or (ii)
		$\alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}$ or $\alpha+\beta+\frac{2}{3}(\alpha+\beta)$	M1		Express sum of new roots in terms of $\alpha+\beta$ and $\alpha\beta$
			M1		Substitute their values into their expression
		$p=\frac{5}{6}$	A1	**4**	Obtain **given** answer correctly
		Or			
		$3u^2-u+2(=0)$	B1		Substitute $x=\frac{1}{u}$ and obtain correct quadratic (equation)
			M1		Use sum of roots of new equation
			M1		Substitute their values into their expression
		$p=\frac{5}{6}$	A1		Obtain **given** answer correctly

(ii)	$\alpha'\beta' = \alpha\beta + \frac{1}{\alpha\beta} + \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$	B1		Correct expansion
	$\frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$	M1		Show how to deal with $\alpha^2 + \beta^2$
		A1		Obtain correct expression
	$q = \frac{1}{3}$	M1		Substitute their values into $\alpha'\beta'$
		A1	**5**	Obtain correct answer a.e.f.
		9		

9 **(i)**		M1		Show correct expansion process for 3 x 3
		M1		Correct evaluation of any 2 x 2
	det **M** = $a^2 - 7a + 6$	A1	**3**	correct answer
(ii)		M1		Solve det**M** = 0
	a = 1 or 6	A1A1	**3**	Obtain correct answer, ft their (i)
(iii)		M1		Attempt to eliminate one variable
		A1		Obtain 2 correct equations in 2 unknowns
		A1	**3**	Justify infinite number of solutions **SC** 3/3 if unique solution conclusion consistent with their (i) or (ii)
		9		

10 **(i)**		M1		Use correct denominator
		A1	**2**	Obtain **given** answer correctly
(ii)		M1		Express terms as differences using (i)
		M1		Do this for at least 3 terms
		A1		First 3 terms all correct
		A1		Last 2 terms all correct
	$\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$	M1		Show relevant cancelling
		A1	**6**	Obtain correct answer a.e.f.
(iii)	$\frac{1}{2}$	B1ft		S_∞ stated or start at n + 1 as in (ii)
	$\frac{1}{n+1} - \frac{1}{n+2}$	M1		S_∞ - their (ii) or show correct cancelling
	$\frac{1}{(n+1)(n+2)}$	A1	**3**	Obtain **given** answer correctly
		11		